Calculation of the gravitational constant G

The Law of Gravity states that all masses attract each other, that this force is proportional to the product of the masses, and indirectly proportional to the square of the distance between the masses. The gravitational constant makes an equation out of this relationship; it is one of the important universal constants of nature. In order to determine the constant, which was also named after Newton, exclusively mechanical methods were used in the past, which are subject to many disturbance variables, which is why G has hitherto been regarded as a constant with great inaccuracy.

By comparison, the uncertainty of a calculation is only due to the accuracy of the natural constants used. In the past, there were several fruitless attempts to compute G [1]. In the following, the author shows on the basis of the theory created in [2], [3]. Fundamentals a new possibility for the mathematical determination of the gravitational constant.

Calculations bring a reduction of the uncertainty of G by several orders of magnitude comes within reach. First of all, the determination by means of two opposing electrons at a distance r appears obvious. Analogous to Coulomb's law, in which charges with different signs attract each other, gravitation can be understood as an attraction of opposite poles, whereby Coulomb force F_c and gravitational force F_q differ greatly in magnitude:

$$F_c = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r^2} = 2.307077 \times 10^{-22} N$$

$$F_g = G\frac{m_e^2}{r^2} = 5.536972 \times 10^{-65} N$$

$$\frac{F_c}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0} \times \left(\frac{e}{m_e}\right)^2}{G} = 4.166 \times 10^{42}$$

The quotient $\frac{F_c}{F_g}$ is also called Eddington's number and even for Feynman the balance of forces between two interacting electrons had great significance. The quotient $\frac{F_c}{F_g}$ can be replaced by the term $\frac{N^2}{24}$, wherein N is referred to as a large number [2]. First of all, it is assumed that the $G=6.67259(85)\times10^{-}11[\frac{m^3/kg}{s^2}]$, recognized by the International Codata Committee in 1986, [4].

$$\frac{F_c}{F_g}=\frac{N^2}{24}=\frac{\frac{1}{4\pi\,\varepsilon_0}\times(\frac{e}{m_e})^2}{G}$$

$$N=\sqrt{\frac{24\times(\frac{e}{m_e})^2}{4\,\pi e_0 G}}=1.000001155\times10^{22} \quad (1)$$

The result indicates that the Large Number N is a large number of about 1×10^{22} . In 1986, data such as $G=6.672605\times10^{-11}$ was typical, with the $N=1\times10^{22}$ large number being assumed without further explanation [3]. The relationship between mass and natural constants is defined by the following relationship, Planck mass corresponding to M_o without $\frac{\pi}{2}$ Planck s intention of 1900. Z_0 is the characteristic impedance of the vacuum [3].

$$kg=10^7\frac{A}{V}\times\frac{2\alpha}{3}\times M_o Z_0$$

These Eqs. connects to other constants of nature, and its extension offers the possibility of determining G_0 as the basis for baseline data to determine the large number N_0 . In the following, the letter b is used for the fine structure constant α .

$$\frac{kg/m}{s}=4\,\pi\times\frac{2}{3}bc\times\sqrt{\frac{hc}{G}}$$

$$(\frac{kg\times m}{s})^2=(4\,\pi\times\frac{2}{3}b\,c)^2\times\frac{hc}{G}$$

$$G_0=\frac{64}{9}\times\pi^2 b^2 h\frac{c^3}{(\frac{kg\times m}{s})^2}=6.672460436911\times10^{-11} \quad (2)$$

$$G_0=\frac{\frac{m^2\times kg}{s}\times\frac{m^3}{s^3}}{(\frac{kg\times m}{s})^2}=[\frac{m^3/kg}{s^2}]$$

The index 0 at G_0 refers to the value for further calculations. After transformation of Eq. (1), one obtains

$$G=\frac{24}{4\pi e_0}\times\frac{(\frac{e}{m_e})^2}{N^2}$$

And by substitution of $e^2=4\pi e_0 c^2 m_e r_e$, the absolute minimum variant of an equation for calculating G arises from this:

$$G=24c^2\frac{r_e}{m_e N^2}[\frac{m^3/kg}{s^2}] \quad (3)$$

By equating equation (3) with equation (2), N can be obtained:

$$\frac{c^2 r_e}{m_e}\times\frac{24}{N^2}=\frac{\frac{64}{9}\times\pi^2 b^2 h c^3}{\frac{kg\times m^2}{s}}$$

$$N_0=\frac{\frac{3}{8}\times\sqrt{\frac{24 r_e}{c h m_e}}}{\pi b}\times\frac{kg\times m}{s}$$

$$N_0=\frac{\frac{3}{4}}{\pi b}\times\sqrt{\frac{6 r_e}{c h m_e}}\times\frac{kg\times m}{s}=1.000010863884\times10^{22} \quad (4)$$

$$N_0=\sqrt{\frac{m\times\frac{s}{m}\times\frac{\frac{s}{kg}}{m^2}}{kg}}\times\frac{kg\times m}{s}=dimensionless$$

The index at N_0 indicates the basis for further calculations. The rel. Uncertainty of N_0^2 is due to the uncertainties of the constants involved with $r_e\pm6.8\times10^{-10}$, $h\pm1.2\times10^{-8}$ and

$m_e \pm 1.2\times10^{-8}$, the sum of which is $\pm 2.5\times10^{-8}$. Codata published the following binding values for G during the period from 1986 to 2014:

Table I

Codata value	Uncertainty	Date
$G_1 = 6.67259\times10^{-11}$	$\pm 1.3\times10^{-4}$	1986
$G_2 = 6.67300\times10^{-11}$	$\pm 1.5\times10^{-3}$	1998
$G_3 = 6.67420\times10^{-11}$	$\pm 1.5\times10^{-4}$	2002
$G_4 = 6.67428\times10^{-11}$	$\pm 1.0\times10^{-4}$	2006
$G_5 = 6.67384\times10^{-11}$	$\pm 1.2\times10^{-4}$	2010
$G_6 = 6.67408\times10^{-11}$	$\pm 4.7\times10^{-5}$	2014

The data show that G reached a maximum in 2006, which flattened again in the following years. The idea was born to relate this decreasing trend to the electron mass m_e , which, according to the transformed equation (3), results in G , so $m_e = 24\dfrac{c^2 r_e}{G N^2}$. These values were referenced to the values G_0 and N_0 given in Eq. (2) and Eq. (4) for a precise evaluation. This results in the following deviations for the individual annual values:

Table II

$G_0 = 6.6724604 \times 10^{-11}$	m_e -related deviations	
$Abw = \frac{\frac{c^2 r_e}{G_0} \times \frac{24}{N_0^2}}{m_e}$	= 0.000	Regard
$Abw = \frac{\frac{c^2 r_e}{G_1} \times \frac{24}{N_0^2}}{m_e}$	= -0.194×10^{-4}	1986
$Abw = \frac{\frac{c^2 r_e}{G_2} \times \frac{24}{N_0^2}}{m_e}$	= -0.808×10^{-4}	1998
$Abw = \frac{\frac{c^2 r_e}{G_3} \times \frac{24}{N_0^2}}{m_e}$	= -2.606×10^{-4}	2002
$Abw = \frac{\frac{c^2 r_e}{G_4} \times \frac{24}{N_0^2}}{m_e}$	= -2.726×10^{-4}	2006
$Abw = \frac{\frac{c^2 r_e}{G_5} \times \frac{24}{N_0^2}}{m_e}$	= -2.067×10^{-4}	2010
$Abw = \frac{\frac{c^2 r_e}{G_6} \times \frac{24}{N_0^2}}{m_e}$	= -2.426×10^{-4}	2014

From these deviations related to the electron mass, it was deduced that the G increase indicated by Codata is also due to a m_e decrease given by Eq. (3). It was difficult to find the only cause in question. Between 1986 and 2014, the masses reported by Codata for me were negligibly reduced by $\frac{dm_e}{m_e} = -6.74 \times 10^{-7}$. So it could only be a cardinal error in determining G around 1986, which will be eliminated over the years. The first indications came from the influence of the co-movement of the proton in the hydrogen atom, which is much discussed in specialist literature.

A finite heavy nucleus moves under the influence of the mass of the electron around the common center of gravity, resulting in the Rydberg constant R_y correction of the

$R_{y.real} = \frac{R_y}{1+\frac{m_e}{m_H}} = -5.4\times10^{-4}$ form. At the same time the mass of the electron increases by

its relativist. Orbital velocity $\frac{v}{c} = 3.6\times10^{-3}$, which shows the influence of other variables that are incomplete to be detected [5].

The reduction of the Rydberg constant from R_y to R_{yr} also applies to the mass of the

electron, as well as $\frac{m_{e.real}}{m_e} = \frac{1}{1+\frac{m_e}{m_H}} = -5.4\times10^{-4}$. To determine the effective Rydberg

constant R_{yr} , the hydrogen transition frequency mentioned by Codata in [6] is used:

$v_H(\frac{1S_1}{2} - \frac{2S_1}{2}) = 2.4660614131870\times10^{15} \pm 4.2\,Hz$ (71). This results in the real effective Rydberg constant R_{yr} :

$$R_{yr} = 2.4660614131870\times10^{15}\frac{Hz}{c}\times\frac{4}{3} = 10967860.58657[\frac{1}{m}]$$

$$R_{yr} = 1.0967860586570\times10^{7} \pm 1.8\times10^{-15}[\frac{1}{m}]$$

$$R_y = 1.0973731568508e{+}7 \pm 5.9\times10^{-12}[\frac{1}{m}]$$

The Rydberg constant R_y is considered to be the most accurate natural constant of all. The dimensionless quotient Q between it and the real effective Rydberg constant is the linchpin of the calculation

$$Q = \sqrt{\frac{R_{yr}}{R_y}} = 0.9997324625913 \pm 3.0\times10^{-12} \quad (5)$$

The difference $Q=\sqrt{\frac{R_{yr}}{R_y}}-1=-2.675374\times10^{-4}$ corresponds to the m_e - related deviation shown in Table II. The comparison shows that the quotient can be used to overcome the problems mentioned in reproducing correct ratios for the H atom and also in determining G according to equation (3). On this basis, the gravitational constant can be calculated using equations (3), (4), (5).

$$N_0=1.000010863884\times10^{22}$$
$$N_0^2=(1.000010863884\times10^{22})^2\pm2.5\times10^{-8}$$
$$Q=0.9997324625913\pm3.0\times10^{-12}$$

$$G=24\,c^2\frac{r_e}{m_e N^2}$$
$$G_{neu}=24\,c^2\frac{r_e}{m_e Q N_0^2}=6.67424604\times10^{-11}[\frac{m^3/kg}{s^2}] \quad (6)$$

Now it is investigated which deviations have reliable data for G from the literature compared to the calculated value G_{neu} . To this end, in Table III only credible values are used for comparison, even if they are sometimes several years ago:

Table III

Nominal value	Uncertainty	Ancestry	Year	Source
$G_a=6.67425\times10^{-11}$	$\pm1.26\times10^{-5}$	G World	1997	[7]
$G_b=6.674215\times10^{-11}$	$\pm1.38\times10^{-5}$	Uni Washington	2000	[7], [4]
$G_c=6.67435\times10^{-11}$	$\pm1.9\times10^{-5}$	UCI-14 Input	2014	[6]
$G_d=6.67408\times10^{-11}$	$\pm4.7\times10^{-5}$	Codata values	2014	[6]

As the following Table IV shows, all deviations of these values from the calculated value $G_{neu}=6.67424604\times10^{-11}$ are within the uncertainty stated by the authors. The ratio of calculated deviation to uncertainty is less than one standard deviation.

Table IV

Quotient	Deviation	Dev./Uncertainty
$\frac{Ga}{G_{neu}}-1$	$+5.922\times10^{-7}$	0.047
$\frac{Gb}{G_{neu}}-1$	-4.651×10^{-6}	0.337
$\frac{Gc}{G_{neu}}-1$	$+1.557\times10^{-5}$	0.797
$\frac{Gd}{G_{neu}}-1$	-2.487×10^{-5}	0.529

A comparison of the results of the state-of-the-art data according to [7], Table 7.5 with G_{neu} , shows that all tolerances specified therein are observed. It is different with regard to G given by Codata in [6] Table XV, where of the 14 references only the following 5 have the error tolerances assigned to them:

Bagley and Luther (1997) LANL-97	$\frac{6.67398\times10^{-11}}{G_{neu}}-1=-3.98\times10^{-5}$
Gundlach and Merkowitz (2000, 2002)	$\frac{6.674255\times10^{-11}}{G_{neu}}-1=+1.34\times10^{-6}$
Kleinvoß, Kleinvoß et al. (2002)	$\frac{6.67422\times10^{-11}}{G_{neu}}-1=-3.90\times10^{-6}$
Schlamminger et al. (2006) UZur-06	$\frac{6.67425\times10^{-11}}{G_{neu}}-1=+5.92\times10^{-7}$
Newman et al. (2014) UCI-14	$\frac{6.67435\times10^{-11}}{G_{neu}}-1=+1.55\times10^{-5}$

Table XV shows that values given by Codata to G are partly subject to large variations. This becomes clear when using the mean of the 14 contained values with $G=6.673671\times10^{-11}$ instead of G_{neu} , where only 4 out of 14 values are in the specified tolerance range.

For the practical calculation of G , it makes sense to combine the results given in equation (4) and equation (5) into a constant K .

$$K=(1.000010863884\times10^{22})^2\times0.9997324625913$$

$$K=N_0^2Q=9.997541846643\times10^{43}\pm2.5\times10^{-8} \quad (7)$$

From the abs. Minimal variant Eq. (3) allows further equations to be deduced by replacing the electron radius r_e once by the Relationship $r_e = b\frac{L_c}{2\pi}$ and the other by the relation $r_e = \frac{b^3}{4\pi R_y}$:

$$G_{neu} = 24c^2 \frac{r_e}{m_e K} = 6.67424604\times10^{-11} \pm 3.7\times10^{-8} [\frac{m^3/kg}{s^2}]$$

$$G_{neu} = \frac{12bc^2 \times \frac{L_c}{\pi m_e}}{K} = 6.67424604\times10^{-11} \pm 3.7\times10^{-8} [\frac{m^3/kg}{s^2}]$$

$$G_{neu} = \frac{\frac{6c^2 b^3}{\pi m_e R_y}}{K} = 6.67424604\times10^{-11} \pm 3.7\times10^{-8} [\frac{m^3/kg}{s^2}]$$

The uncertainty in the calculation of G is mainly determined by N_0^2 according to equation (4) or the constant $K \pm 2.5\times10^{-8}$ according to equation (7). In addition, the uncertainty of the electron mass $m_e \pm 1.2\times10^{-8}$ is given in equation (3). The inaccuracy of the constants still involved can be neglected. ($b \pm 2.3\times10^{-10}$, $L_c \pm 4.5\times10^{-10}$, $r_e \pm 6.8\times10^{-10}$). The uncertainty of the electron mass m_e thus determines the total error of the G calculation.

The actual cause of the inaccuracy of m_e is the underlying Avogadro constant with $N_A \pm 1.2\times10^{-8}$, which limits a further increase in the accuracy of G . In principle, the accuracy of G is limited to 3 times the N_A uncertainty. Due to SI efforts, approximation to $N_A = 6.022140758\times10^{23} \pm 1.0\times10^{-8} [\frac{1}{mol}]$ is planned [8], which indicates future accuracy limits of G .

THE G-FIELD IS ENERGY! Consequently, there is no "empty" space. EVEN THE ELECTRON DELIVERS HIS CONTRIBUTION ..... (M. Geilhaupt)

The calculations are based on the following constants:

$$b = 7.297352568653 \times 10^{-3} [\alpha]$$

$$c = 2.99792458 \times 10^{8} [\frac{m}{s}]$$

$$h = 6.626070153139 \times 10^{-34} [J \times s]$$

$$L_c = 2.426310238167 \times 10^{-12} [m]$$

$$m_e = 9.109383707749 \times 10^{-31} [kg]$$

$$N_A = 6.022140758 \times 10^{23} [\frac{1}{mol}]$$

$$r_e = 2.817940325365 \times 10^{-15} [m]$$

$$R_y = 1.097373156852 \times 10^{7} [\frac{1}{m}]$$

Literature

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